

Resonance in Forced Oscillations of an Accretion Disk and Kilohertz Quasi-Periodic Oscillations

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ABSTRACT

We have performed numerical simulations of a radially perturbed “accretion” torus around a black hole or neutron star and find that the torus performs radial and vertical motions at the appropriate epicyclic frequencies. We find clear evidence that vertical motions are excited in a nonlinear resonance when the applied perturbation is periodic in time. The strongest resonant response occurs when the frequency difference of the two oscillations is equal to one-half the forcing frequency, precisely as recently observed in the accreting pulsar, SAX J1808.4-3658, where the observed kHz QPO peak separation is half the spin frequency of 401 Hz.

Subject headings: Stars: neutron — X-rays: binaries — accretion disks — hydrodynamics

1. Introduction

Millisecond oscillations in the X-ray light curves of systems known to contain neutron stars or possibly black holes have been observed for several years with the Rossi X-Ray Timing Explorer (see van der Klis (2000) for a review). Recently, it has been pointed out that the centroid frequencies of the corresponding kilohertz quasi periodic oscillations (kHz QPOs) are in rational ratios of small integers, as 3:2 (Abramowicz & Kluźniak 2001; Remillard et al. 2002; McClintock & Remillard 2003; Abramowicz et al. 2003a,b). This supports suggestions that a resonance of some kind is responsible for the observed properties (Kluźniak & Abramowicz 2001, 2003; Titarchuk 2002). In addition, for at least one system in which coherent pulsations have been detected, indicating the spin

frequency ($\nu_s = 401$ Hz in SAX J1808.4-3658), the separation in frequency between the kHz peaks has been recently reported to be consistent with $\nu_s/2$ (Wijnands et al. 2003), implying that the pulsar is exciting motions in the accretion disk in a nonlinear fashion (Kluźniak et al. 2003).

In this Letter we show that the kHz oscillations detected in SAX J1808.4-3658 can be attributed to forcing of epicyclic motions in the accretion disk by the 2.5 ms pulsar, which induces resonance at selected frequencies. The coupling between the pulsar and the disk could be due to the magnetic field, or to some structure on the surface of the star. In other, similar systems, a frequency separation equal to the stellar spin frequency is also possible.

2. Response of a torus to an external perturbation

In order to study the response of an accretion disk to an external perturbation, we make several simplifying assumptions. The first is to consider not a full accretion disk but a slender torus in hydrostatic equilibrium, orbiting a central body of mass M . By virtue of the pressure effects, the rotation curve is not Keplerian, and for all the cases studied here, we consider constant distributions of specific angular momentum within the torus, which has low mass and is slender in the sense that its mass $m \ll M$, and its extension $L \ll R$, where R is the distance separating it from the mass M . We thus neglect the self-gravity of the torus, which is ellipsoidal in cross section in a meridional slice. We additionally assume azimuthal symmetry.

The second assumption is to use a potential for the central mass $\Phi_{KL} = M[1 - \exp(r_{ms}/r)]/r_{ms}$ (Kluźniak & Lee 2002), which reproduces two features of Einstein’s gravity that are relevant in our context: (1) there is a marginally stable orbit for test masses in circular orbits at $r_{ms} = 6M$, and (2) the vertical and radial epicyclic frequencies (in the equatorial plane) are given by $\zeta^2(r) = (1/4\pi)^2(M/r^3) \exp(r_{ms}/r)$ and $\kappa^2(r) = (1/4\pi)^2(M/r^3) \exp(r_{ms}/r)(1 - r_{ms}/r)$, respectively, so that their ratio is $\kappa(r)/\zeta(r) = (1 - r_{ms}/r)^{1/2}$, coinciding with the exact solution in the Schwarzschild metric in general relativity.

In hydrostatic equilibrium, pressure and centrifugal support balance gravity in the radial direction, and pressure alone balances gravity vertically. An ideal equation of state for the fluid closes the system of equations (we have used an adiabatic index $\gamma = 4/3$ throughout). The center of the torus can be conveniently defined as the locus of maximum density, where the rotation velocity is Keplerian, and its position is denoted by r_0 .

Clearly a detailed answer to the general problem being explored here requires modeling of the full accretion disk, whether it is geometrically thin or thick. We will explore this in future work. For the time being, analyzing the behavior of such a slender torus can be considered analogous to splitting the disk into thin annuli and investigating the properties of one such ring — as in analy-

sis of radiation-induced warping (Petterson 1977; Pringle 1996). The slender torus can be thought of as a density enhancement in the accretion disk, the properties of which might be transmitted and/or amplified into the overall X-ray light curve observed from the system by an unspecified mechanism.

The actual tori are constructed from the specified gravitational potential and equation of state in standard fashion (see e.g. (Igumenshchev, Chen & Abramowicz 1996; Zurek & Benz 1986)), and their dynamical behavior (see below) is followed using a two-dimensional smooth particle hydrodynamics (SPH) code (Monaghan 1992; Lee & Ramirez-Ruiz 2002) that uses cylindrical coordinates (r, z) . A cross section (in density) of an equilibrium configuration thus realized is shown in Fig. 1. We now detail the results obtained from using two different types of perturbations to such an initial state.

2.1. Impulsive perturbation

As a first example, we consider an impulsive perturbation to the torus, analogous to that considered recently by Zanotti, Rezzolla & Font (2003). (We note that these modes might also be relevant for kHz QPOs in black hole systems; see Rezzolla, Yoshida, Maccarone & Zanotti (2003).) The equilibrium torus is given a perturbed velocity field at $t = 0$ and allowed to oscillate freely thereafter. The velocity field of the perturbation is purely radial (in a cylindrical sense). The kick is small, so that the induced oscillations in the radial direction have an amplitude that is of the same order as the extent of the torus itself. The torus performs small oscillations in the radial direction, primarily at a frequency consistent with the local epicyclic frequency κ_0 , as would be expected. However, small vertical motions are also induced owing to pressure coupling between both modes, and these can be clearly seen in the vertical oscillations of the center of the torus, occurring with greatest power at a frequency that is consistent with the local vertical epicyclic frequency, ζ_0 . Both the radial and vertical oscillations are shown in Fig. 2, along with their Fourier transforms.

It is thus clear that if radial motions of some kind are present in the torus, they can induce vertical oscillations, and both of these occur at the epicyclic frequencies for test particles in nearly circular orbits.

2.2. Periodic perturbation

We now consider a situation in which there is a periodic perturbation at work, which is external to the disk. The pulsar at the center of the gravitational potential well is clearly an example of this, and one would expect it to have an effect on the disk. The magnetic field of the pulsar, or some deformation on its surface, can perturb the disk at intervals given by the inverse of the spin period, $\Delta T = 1/\nu_s$ (in the case of SAX J1808.4-3658, $\nu_s=401$ Hz). We have considered then a forcing in the radial direction, which manifests itself through a small, radial acceleration, the magnitude of which is shown in Fig. 3. It is not purely sinusoidal but repeats at a fixed interval ΔT , as given above. The adopted profile of this perturbation deserves some comment: we have used it to mimic the passage of a brief (compared with the repetition time), but fairly stronger than average, disturbance in the accretion disk (e.g., the corresponding polar magnetic field of the pulsar, sweeping around the disk). Different shapes for this pulse will be explored elsewhere.

The radial forcing obviously induces a radial oscillation of the torus. Because of pressure coupling, a vertical oscillation is again apparent, albeit at a reduced magnitude (compared with the radial amplitude). The motion of the center of the torus (as defined above) can be Fourier-analyzed to extract the relevant frequencies. We find that the radial and vertical motions occur primarily at the local radial and epicyclic frequencies, κ_0 and ζ_0 respectively, at r_0 . For a torus orbiting a mass $M = 1.38M_\odot$, and center at $r_0 = 12.25M = 24.8$ km, $\kappa_0 = 500$ Hz and $\zeta_0 = 700$ Hz.

The question now is: what effect, if any, does the repetition time ΔT of the perturbation have on the power of oscillatory motion induced in the torus? The radial motion is always driven at a relatively high amplitude, simply because the perturbation is itself applied as a radial acceleration. We find, however, that the power of the vertical motions varies greatly as ΔT is altered.

We have kept the initial position of the torus fixed at $r_0 = 12.25M$ and performed over a dozen simulations, differing only in the value of ν_s , and covering a range $100 < \nu_s(\text{Hz}) < 600$. For each one of these, we have computed the peak power $P_{M,z}$ in the vertical oscillation at ζ_0 . Since one

could equivalently keep ν_s fixed (as is actually the case in nature) and vary r_0 (which would alter the difference $\zeta_0 - \kappa_0$), in Fig. 4 we show $P_{M,z}$ as a function of the ratio $\nu_s/(\zeta_0 - \kappa_0)$. The response of the torus is clearly greatest when $\nu_s = 2(\zeta_0 - \kappa_0)$, as was observed in the single instance for SAX J1808.4-3658 when two QPO peaks in the kHz range were seen.

We note that there is also a strong response when ν_s and $\zeta_0 - \kappa_0$ are in a 1:1 or 3:2 correspondence. The first of these would allow for the possibility of twin peaks with a separation of 401 Hz in SAX J1808.4-3658, while the second would imply a separation of $802/3 = 267$ Hz. The first option would occur at $\zeta_0=1054$ Hz, $\kappa_0=653$ Hz (see also Kluźniak et al. (2003)), and $r_0=9.75M=19.7$ km for $1.38M_\odot$. The second would imply $\zeta_0=832$ Hz and $\kappa_0=565$ Hz at a radius $r_0=11.13M=22.5$ km for the same mass. Neither has been observed as yet.

3. Discussion

The appearance of twin kHz QPOs in the millisecond pulsar SAX J1808.4-3658, with a separation consistent with half the known spin frequency of the pulsar, strongly indicates that a nonlinear resonance is at work, coupling the spin to vibrational modes in the disk. Using a simple hydrodynamical model, we identify these modes with the radial and epicyclic oscillations of fluid elements slightly displaced from exact circular orbits. In this respect, the model is crucially dependent on the effects of strong gravity, to break the degeneracy between the orbital and epicyclic frequencies present in the Newtonian regime.

Under the unique assumption that the pulsar provides a periodic driving radial force to a slender torus in orbit (which we consider as a stand-in for a density enhancement in the accretion disk; see § 2), we show that the response of the torus is greatest when the spin frequency is twice the difference between the vertical and radial epicyclic frequencies. We also show that there are other possibilities for resonant motion, when the above numbers are in a 1:1 or 3:2 correspondence.

For the case of SAX J1808.4-3658, the frequency ratio $700:500=1.4$, and the actual values of the frequencies observed would allow us to fix the mass of the pulsar at 1.38 solar masses in the

Schwarzschild metric. However, the actual value will be different, as the epicyclic frequencies for a neutron star rotating at 401 Hz depart from the Schwarzschild values (Kluźniak et al. 2003).

Two further points deserve comment in the context of kHz QPOs in systems with a millisecond pulsar whose spin frequency is known. First, there is an apparent dichotomy in the values of the QPO peak separation with respect to the spin frequency. For the “fast” rotators, like SAX J1808.4-3658, the separation is half the spin frequency, $\Delta\nu = \nu_s/2$, while for “slow” rotators, like XTE J1807-294 (where $\nu_s = 190$ Hz), the separation is consistent with the spin frequency, $\Delta\nu = \nu_s$. This could be explained within the current framework, since for fast rotators, the resonant point corresponding to $\Delta\nu = \nu_s$ is so close to the neutron star that it is unlikely to be seen (one could say that there is no room around the pulsar for this mode to occur). For slow rotators, both resonances, at ν_s and $\nu_s/2$, could in principle be observed, since they would manifest themselves at greater distances from the neutron star. However, if the modulations in flux generated in the innermost regions of the accretion disk have a dominant effect on the overall X-ray light curve, one would then expect to see the strongest signal at frequency ν_s .

Second, the twin kHz QPOs then need not necessarily occur always at the same frequencies, as the torus shifts its radial position, but their separation should nevertheless remain roughly constant (and equal to ν_s or $\nu_s/2$, depending on the system) under this excitation mechanism, assuming mode locking to occur. Further timing observations of millisecond pulsars will no doubt shed light on these matters.

More generally, our simulation shows directly that a radial perturbation present in the torus can excite forced oscillation at (other) eigenfrequencies. This may have implications for mode coupling in black hole accretion disks.

Financial support for this work was provided in part by CONACyT (36632E) and by the Polish Committee for Scientific Research (KBN grant 2P03D01424). We thank the referee for his comments on the original version of the manuscript.

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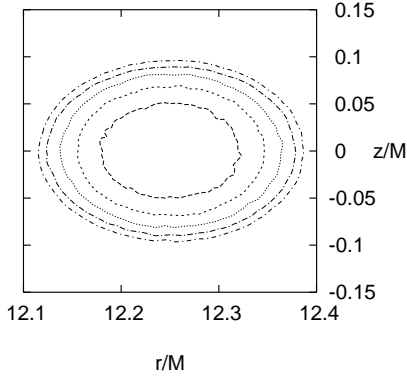


Fig. 1.— Logarithmic density contours in a meridional slice of a slender torus in hydrostatic equilibrium, spaced every 0.2 dex, with the lowest one at $\log(\rho/\rho_{\max}) = -1$. The center of the torus is at $r_0 = 12.25$ M.

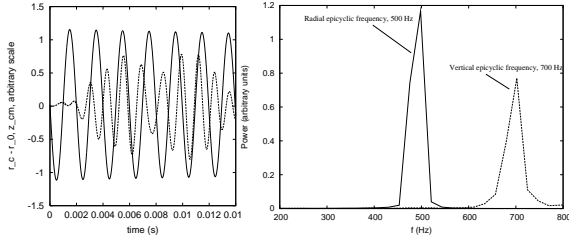


Fig. 2.— Left: Radial (solid line) and vertical (dashed line) oscillations performed by the center of the torus after an impulsive radial perturbation at $t = 0$. Right: Fourier transforms of the radial (solid line) and vertical (dashed line) oscillations shown in the left panel. The two curves have been rescaled so as to fit on the same graph.

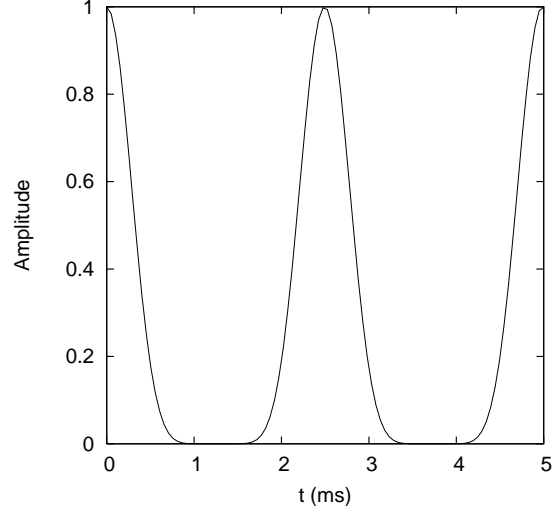


Fig. 3.— Amplitude of the radial perturbation (normalized to the maximum value) applied to the torus, with a repetition frequency of $\nu_s = 401$ Hz.

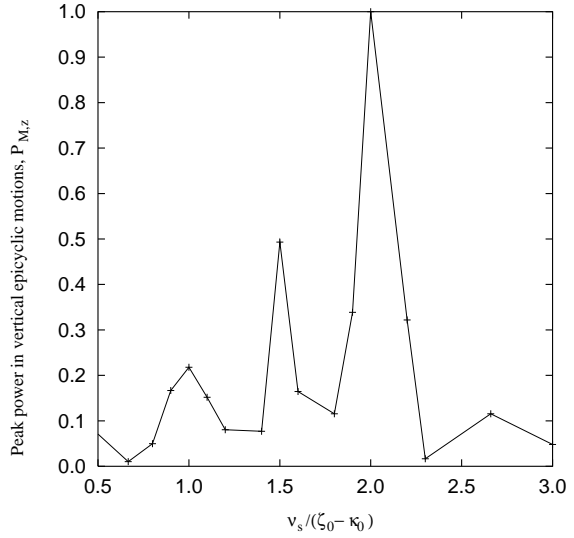


Fig. 4.— Peak power in the vertical epicyclic oscillations induced in the torus by the periodic perturbation of Fig. 3, as a function of $\nu_s/(\zeta_0 - \kappa_0)$. Three peaks at 1.0, 1.5, and 2.0 are clearly visible. For this calculation, ν_s , the pulsar perturbation frequency, varied while $\zeta_0 - \kappa_0$ was held fixed at 200 Hz.